Sec 6.5 Quadratic Equations

Has an x^2 term, but no higher power.

$3x + 5$

terms

$3x$ is also called a <u>mo</u>nomial.

$3x + 5$ is a <u>b</u>inomial.

$x+2$ is also a binomial.

what happens when we multiply two binomials.

$(3x+5)(x+2)$ Distributive property gives

$(3x+5)x + (3x+5)\cdot 2$ Distributive property again

$(3x)x + 5x + 3x\cdot 2 + 5\cdot 2$

product of <u>last</u> terms

product of outer terms

product of inner terms

product of first terms

$3x^2 + 5x + 6x + 10$

$= 3x^2 + 11x + 10$ quadratic expression

The FOIL method

First Outer Inner Last mnemonic for result of multiplication of 2 binomials.

$(2x-3)(x+5)$

$(2x)x - 3x + (2x)(5) - 3(5)$

$2x^2 - 3x + 10x - 15$

$2x^2 + 7x - 15$

Notice that $2x-3$ and $x+5$ are <u>factors</u> of $2x^2+7x-15$ <u>Trinomial</u>

Factoring a Trinomial whose squared term coefficient is 1.

$x^2 + 4x + 3$

$(x \quad)(x \quad)$

has to be x

Have to be factors of 3 which sum to 4.

$$(x \quad 1)(x \quad 3)$$

have to be +

$$(x+1)(x+3) = x^2 + 4x + 3$$

just like $2 \cdot 3 = 6$.

Example: Factor $x^2 + 2x - 8$

List pairs of factors of 8

$-1, 8$ $\quad$ $1, -8$

$-2, 4$ $\quad$ $2, -4$

Find a pair whose sum is $+2$

$-2 + 4 = 2$

$$(x-2)(x+4)$$

Class do : Factor $x^2 - 6x - 7$.

Factors of -7 $1, -7$ $-1, 7$

$(x+1)(x-7) = x^2 - 6x - 7$

Class do : Factor $x^2 + 8x + 15$

$1, 15$ $-1, -15$

$3, 5$ $-3, -5$

which pair sums to eight ?

$(x + 3)(x + 5) = x^2 + 8x + 15$

Sec 6.5

Factoring when leading coefficient is not 1.

Example: Factor $3x^2 + 5x - 28$

Pair of binomials must look like

$(\_\ x \qquad)(\_\ x \qquad)$

factors of 3 $\qquad$ 1 $\quad$ 3

$(3x \qquad)(x \qquad)$

List Factors of -28

$-1, 28$	$-2, 14$	$-4, 7$
$1, -28$	$2, -14$	$4, -7$

Possible factorizations	sum of outside & inside products [Need 5]
$(3x - 1)(x + 28)$	83
$(3x + 28)(x - 1)$	25
$(3x + 1)(x - 28)$	-83
$(3x - 28)(x + 1)$	-25
$(3x - 2)(x + 14)$	40
$(3x + 14)(x - 2)$	8

$(3x+2)(x-14)$ -40

$(3x-14)(x+2)$ -8

$(3x-4)(x+7)$ 17 -2 -14

$(3x+7)(x-4)$ -5

$(3x+4)(x-7)$ -17

$(3x-7)(x+4)$ 5 ← The one we need.

Example: After Practice:

Factor $5x^2 - 24 + 28$

$(5x-14)(x-2)$

1 28
2 14
4 7

Talk throug Signs

Sec 6.5

solving quadratic equations by factoring:

$ax^2 + bx + c = 0$ with $a \neq 0$

is a quadratic equation.

Zero product Principal: If

$ab = 0$, then $a = 0$ or $b = 0$

or both.

zero is special in this way.

Pg 356 List of steps.

Example: Solve

$x^2 = -x + 6$

$x^2 + x - 6 = 0$

$(x + 3)(x - 2) = 0$

$x + 3 = 0$ or $x - 2 = 0$

$x = -3$ or $x = 2$

Sec 6.5

Example: Solve by factoring

$$5x^2 + 54x + 40 = 0$$

1, 5 | 1 40, 2 20, 4 10

$$(5x + 4)(x + 10) = 0$$

$5x + 4 = 0$ or $x + 10 = 0$

$5x = -4$ $\quad$ $x = -10$

$x = \frac{-4}{5}$

$\frac{16}{5} + \frac{216}{5} + \frac{200}{5}$

$$\left\{ -\frac{4}{5}, -10 \right\}$$

Solving with the Quadratic Formula

The solutions of $ax^2 + bx + c = 0$, with $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

1. Two numbers

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

2. Zero on one side & Sort it.

Example: Solve using quadratic formula

$$x^2 = -x + 6$$

$$x^2 + x - 6 = 0$$

$a = 1$
$b = 1$
$c = -6$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-6)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{25}}{2}$$

$$\frac{-1+5}{2} \quad \text{or} \quad \frac{-1-5}{2}$$

$$2 \quad \text{or} \quad 3$$

$x = 2$ or $x = -3$

Solution set $\{2, -3\}$

Example: Solve Using the Q.F.

$$5x^2 + 54x + 40 = 0$$

2116

$$x = \frac{-54 \pm \sqrt{(54)^2 - 4(5)(40)}}{2(5)}$$

$$x = \frac{-54 \pm \sqrt{2116}}{10}$$

$$= \frac{-54 \pm 46}{10}$$

$$= \frac{-100}{10} \quad \text{or} \quad \frac{-8}{10}$$

$$= -10 \quad \text{or} \quad -\frac{4}{5}$$

$$\left\{-\frac{4}{5}, -10\right\}$$

Example: Solve using the quadratic formula.

$$2x^2 = 8x - 5$$

$$2x^2 - 8x + 5 = 0$$

$a = 2$

$b = -8$

$c = 5$

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(2)(5)}}{2(2)}$$

$$= \frac{8 \pm \sqrt{64 - 40}}{4}$$

$$= \frac{8 \pm \sqrt{24}}{4}$$

$$= \frac{8 \pm 2\sqrt{6}}{4}$$

$$\left\{ 2 + \frac{\sqrt{6}}{2},\ 2 - \frac{\sqrt{6}}{2} \right\}$$

0.775255

3.224745

Example 12 on page 359 gives this formula for the normal systolic blood pressure P of a man age A.

Blood pressure P
Age A

$$P = 0.006A^2 - 0.02A + 120$$

Find the age at which normal Blood pressure is 135 mm. Hg..

$$135 = 0.006A^2 - 0.02A + 120$$

$$0 = 0.006A^2 - 0.02A - 15$$

$$A = \frac{0.02 \pm \sqrt{(0.02)^2 - 4(0.006)(-15)}}{2(0.006)}$$

$$= \frac{0.02 \pm \sqrt{0.0004 + 0.36}}{0.012}$$

$$= \frac{0.02 \pm \sqrt{0.3604}}{0.012}$$

51.69 or -48.36

About 52 years.