5.5 Real Numbers and Their Properties

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Recall that we have studied the

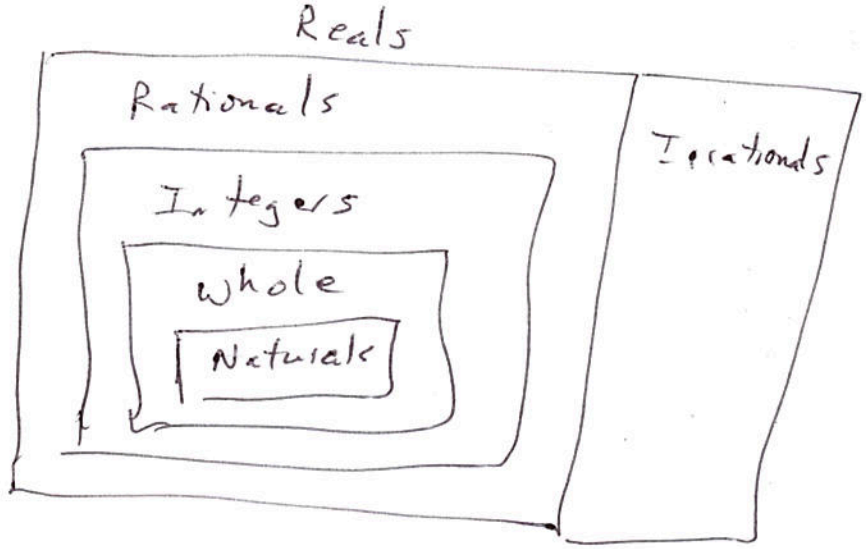
Natural Numbers $\{1, 2, 3, \ldots\}$

Whole Numbers $\{0, 1, 2, 3, \ldots\}$

Integers $\mathbb{Z}$ $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$

Rational Numbers $\mathbb{Q}$ $\left\{ \frac{a}{b} : a \text{ and } b \text{ are integers and } b \neq 0 \right\}$

Irrational Numbers decimal representations neither terminate nor repeat.



Subsets of the real numbers

All are on the numberline, and everything on the line is one of these

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Example:

Consider the set $\left\{ 18, -5, \frac{7}{12}, \pi, 3\sqrt{2}, \frac{-4}{15} \right\}$

which are

whole? 18,

Integers? 18, -5

Rational? $18, -5, \frac{7}{12}, \frac{-4}{15}$

Irrational? π, $3\sqrt{2}$

Properties of Real Number Arithmetic: For real numbers a, b, c

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Additive property of

closure $a+b$ is real $\quad 3\sqrt{2} + (-3\sqrt{2})$ is not irrational

commutativity $a+b = b+a$

associativity $(a+b)+c = a+(b+c)$

identity $0+a = a = a+0$

inverse $a+(-a) = 0$ $\quad -a$ is <u>there</u>

Multiplicative property of

closure ab is real

commutativity $ab = ba$

associativity $(ab)c = a(bc)$

identity $1a = a \cdot 1 = a$

inverse If $a \neq 0$, then $a\left(\frac{1}{a}\right) = 1$

Distributive property of multiplication over addition

$a(b+c) = ab + ac$ $\quad 7(\sqrt{2}+8) = 7\sqrt{2} + 7 \cdot 8$

Example: Which property is represented?

$3(\sqrt{5}+4) = 3\sqrt{5} + 12$ Dist. prop of mult over addition

$2 \cdot 3 = 3 \cdot 2$ Commut. mult.

$1 \cdot 3 = 3$ Identity mult.

$\frac{1}{2} + (\sqrt{17} + 4) = \left(\frac{1}{2} + \sqrt{17}\right) + 4$ Assoc Add.

$-3\left(\frac{-1}{3}\right) = 1$ Inverse. mult.

Example?

Are the Natural numbers closed under addition? Yes

Are the irrationals closed under multiplication? No

$3\sqrt{2} \cdot \sqrt{2} = 6$