

Section 5.4

Irrational Numbers

Recall that a rational number can be expressed as a ratio of integers $\frac{a}{b}$.

As a consequence, the decimal representation of a rational number either terminates or repeats.

The set of irrational numbers is the set of numbers whose decimal representations neither terminate nor repeat.

Examples

$$0.23223222322223\dots$$

$$\sqrt{2} \approx 1.414214\dots$$

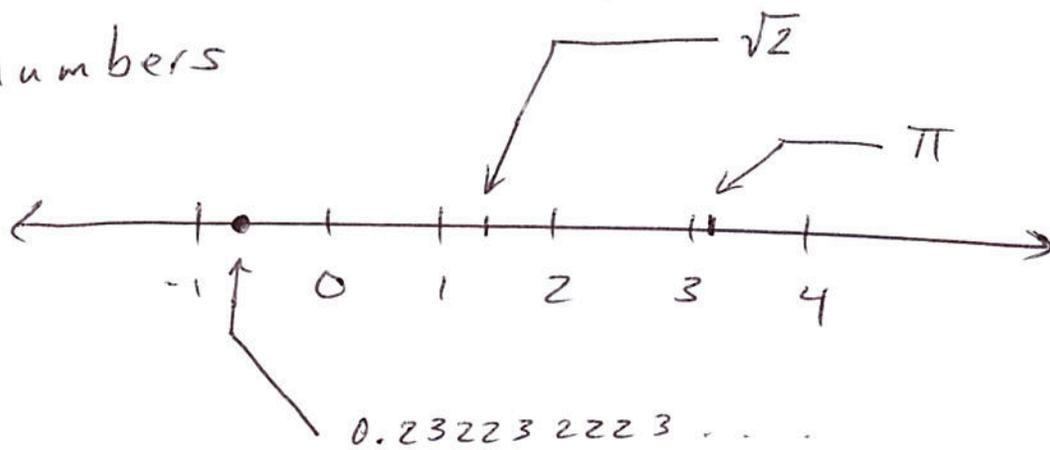
$$\pi \approx 3.141592653589\dots$$

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These are still perfectly good numbers



Square roots are often irrational.

Say The principal square root of a non negative number n , written \sqrt{n} , is the positive number that when multiplied by itself gives n .

$$\sqrt{36} = 6 \quad \text{rational}$$

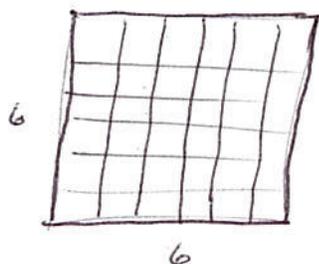
$$\sqrt{49} = 7 \quad \text{rational}$$

$$\sqrt{3} \approx 1.73205 \quad \text{irrational}$$

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36 and 49 are perfect squares:
square root is a whole number.



The square root operation and multiplication commute.

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If a and b represent non negative numbers, then

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

The square root of a product is the product of the square roots.

Say

$$\sqrt{36 \cdot 4} = \sqrt{144} = 12$$

$$\sqrt{36 \cdot 4} = \sqrt{36} \sqrt{4} = 6 \cdot 2 = 12$$

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Simplify
 Say Simplifying a square root means to remove from the square root any perfect squares that occur as factors of the radicand.

Examples: Simplify

$$1) \sqrt{108} = \sqrt{36 \cdot 3} = \sqrt{36} \sqrt{3} = 6\sqrt{3}$$

or

$$\sqrt{108} = \sqrt{2^2 \cdot 3^3} = \sqrt{2^2 \cdot 3^2} \sqrt{3} \\ = 6\sqrt{3}$$

$$2) \sqrt{490} = \sqrt{49 \cdot 10} = \sqrt{49} \sqrt{10} \\ = 7\sqrt{10}$$

Calculate $(7\sqrt{10})^2$ on calculator.

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Example : Multiply

$$\sqrt{3} \sqrt{4} = \sqrt{12}$$

$$\begin{aligned}\sqrt{15} \sqrt{3} &= \sqrt{45} = \sqrt{9 \cdot 5} \\ &= \sqrt{9} \cdot \sqrt{5} \\ &= 3\sqrt{5}\end{aligned}$$

Division also commutes with square root operation.

If a and b are non negative and $b \neq 0$, then

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

$$\frac{\sqrt{108}}{\sqrt{3}} = \sqrt{\frac{108}{3}} = \sqrt{36} = 6$$

$$\frac{\sqrt{90}}{\sqrt{2}} = \sqrt{\frac{90}{2}} = \sqrt{45} = 3\sqrt{5}$$

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do not interact w/ square root
as simply.

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Example: Add

$$4\sqrt{3} + 7\sqrt{3} = 11\sqrt{3}$$

$$8\sqrt{2} + 5\sqrt{7} \quad \text{Done}$$

$$4\sqrt{5} - 6\sqrt{5} = -2\sqrt{5}$$

Some times simplifying allows us
to ~~add or subtract~~. combine surmands

$$\begin{aligned} & 3\sqrt{2} + \sqrt{8} \\ &= 3\sqrt{2} + \sqrt{4 \cdot 2} \\ &= 3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2} \end{aligned}$$

Class work:

$$\begin{aligned} & 4\sqrt{8} - 7\sqrt{18} \\ &= 8\sqrt{2} - 21\sqrt{2} = -13\sqrt{2} \end{aligned}$$

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Rationalizing denominators.

Notice

$$\frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\frac{1}{\sqrt{3}} = \frac{1 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{3}}{3} \quad \leftarrow \text{denominator is rational}$$

Example:

Rationalize the denominator

$$\frac{8}{\sqrt{5}} = \frac{8\sqrt{5}}{5}$$

$$\sqrt{\frac{4}{7}} = \frac{\sqrt{4} \sqrt{7}}{\sqrt{7} \sqrt{7}} = \frac{2\sqrt{7}}{7}$$

Cube and higher roots:

The cube root of n is the number whose cube is n .

$$\sqrt[3]{8} = 2 \quad \text{since } 2 \cdot 2 \cdot 2 = 8$$

 $\sqrt[3]{20}$ is irrational